

Technical Note

Application of a simple finite difference model for estimating evaporation from open water

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Abstract

In estimating the evaporation from open water, the challenge is to accurately quantify the change in heat stored in the water body. A simple finite difference model is described and a comparison made between measured values of water temperature and evaporation, from a reservoir in southeast England, and the values predicted by an equilibrium temperature model. The values predicted by the new model are in excellent agreement with the measurements and are closer to the measured values than those predicted by the equilibrium temperature model. The difference in performance is attributed to improved methods used for calculating the net radiation and the wind function. The simpler formulation of the finite difference model is considered to offset the disadvantage of the greater number of calculations required. © 2002 Elsevier Science B.V. All rights reserved.

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1. Introduction

The management of water resources and freshwater ecosystems increasingly needs estimates of open water evaporation. For example, estimates are needed to support the management of wetlands and other still waters, as well as for the appraisals of applications for abstraction licences. Measurements over the water, of either evaporation rates or the basic meteorological variables, are rarely available and so the generally adopted procedure is to estimate the evaporation using simple models driven by meteorological measurements made over the dry-land surface. The energy balance of water bodies can be different to that of other surfaces because the incoming solar radiation is absorbed within the water column, rather

than at the surface. The heat storage term is therefore likely to be a significant component of the energy balance and there can be a significant lag between the seasonal radiation and evaporation. Thus, to make robust estimates of evaporation, the critical requirement is for an accurate model of the change in heat storage of the water body. This paper describes a simple finite difference model for estimates of heat storage and compares the model results with those of Finch (2001), who evaluated estimates of open water evaporation made using the equilibrium temperature method against measurements of evaporation from a reservoir under controlled management.

2. Theory

The estimation of open water evaporation can be simplified considerably if it is assumed that the water column is well mixed, i.e. there is no thermal

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stratification. This assumption is made by both the models described here.

2.1. The equilibrium temperature method

Edinger et al. (1968) introduced the concept of an equilibrium temperature as a means of quantifying the change in heat storage. The equilibrium temperature is the temperature towards which the water temperature is driven by the net heat exchange, i.e. when the water is at equilibrium temperature, then the net rate of heat exchange is zero. From this, and an associated time constant, he was able to derive an expression for the temperature of a well-mixed body of water as a function of time and water depth. Once the water temperature is estimated, then it can be used to estimate the evaporative and sensible heat fluxes, the heat storage and the outgoing longwave radiation from the water. This concept has been used by a number of other workers, notably Keijman (1974), Fraedrich et al. (1977), de Bruin (1982) and Finch (2001).

The model used in this study has been fully described by Finch (2001) and will not be repeated in detail here. However, the method used to estimate the outgoing longwave radiation is important and so is described. An initial estimate of the outgoing longwave radiation, L^1 ($\text{MJ m}^{-2} \text{d}^{-1}$), is made using the wet bulb temperature, T_n ($^{\circ}\text{C}$)

$$L^1 = p(\sigma(T_a + 273.1)^4 + 4\sigma(T_a + 273.1)^3(T_n - T_a))$$

where p is a cloudiness factor, σ the Stefan–Boltzman constant $= 4.9 \times 10^{-9}$ ($\text{MJ m}^{-2} \text{K}^{-4} \text{d}^{-1}$) and T_a is the air temperature ($^{\circ}\text{C}$) at reference height. This estimate of the outgoing longwave radiation is used in calculating the net radiation, which is in turn used to estimate the equilibrium temperature from which the temperature of the water body at the end of the current time step is calculated. The estimated water temperature is then used to make a second estimate of the outgoing longwave radiation by substituting the estimated water temperature for the wet bulb temperature in the above equation, allowing the evaporation to be calculated from the energy balance.

The model uses an empirical wind function, to give the turbulent exchange coefficients for the transfer of water vapour and sensible heat through the boundary layer between the water surface and reference height. In addition to wind speed, it depends on a number of

factors including the measurement height, surface roughness and the stability of the atmosphere. The wind function selected by Sweers (1976), which was developed from a cooling pond in Wales, is used

$$\lambda f(u) = 0.864(4.4 + 1.82u)$$

where u (m s^{-1}) is the wind speed at a height of 10 m and λ is the latent heat of vaporization ≈ 2.45 (MJ kg^{-1}).

2.2. The finite difference method

Since time series of estimates of open water evaporation are normally required, an alternative to the equilibrium temperature method is a simple finite difference scheme in which the water temperature is estimated by iteration. The net radiation, R_n ($\text{MJ m}^{-2} \text{d}^{-1}$), is calculated as

$$R_n = K^1(1 - \alpha) + L^1 - p\sigma(T_w + 273.1)^4$$

where K^1 is the incoming short-wave radiation ($\text{MJ m}^{-2} \text{d}^{-1}$), α the short-wave albedo of the water surface, L^1 the incoming long-wave radiation ($\text{MJ m}^{-2} \text{d}^{-1}$) and T_w the average water temperature ($^{\circ}\text{C}$) calculated as

$$T_w = T_{w,i-1} + (T_{w,i} - T_{w,i-1})/2$$

and $T_{w,i}$ is the estimated water temperature ($^{\circ}\text{C}$) at the end of the current time step and $T_{w,i-1}$ is the estimated water temperature at the end of the previous time step. The change in heat stored in the water column during the current time step, W ($\text{MJ m}^{-2} \text{d}^{-1}$), is calculated from the energy balance as

$$W = R_n - \lambda E - H$$

where λE is the latent heat flux and H is the sensible heat flux. These have been calculated from the standard flux-gradient equations for a water surface (see Brutsaert, 1982)

$$\lambda E = f(u)(e_w^* - e_d)$$

$$H = \gamma f(u)(T_w - T_a)$$

where the sign convention is positive for fluxes away from the surface, e_w^* is the saturated vapour pressure at the water temperature (kPa), e the average vapour pressure at the reference height (kPa) and γ the psychrometric constant (kPa K^{-1}). Changes in the

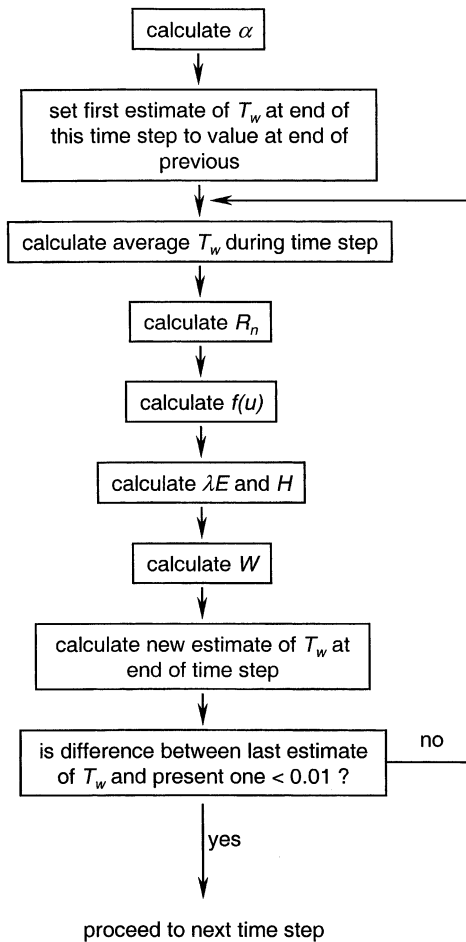


Fig. 1. Flow diagram of the finite difference model for a single time step (α — water surface albedo, T_w — water temperature, R_n — net radiation, $f(u)$ — wind function, λE — latent heat flux, H — sensible heat flux, W — change in heat stored in the water).

stability of the atmosphere overlying the water are accommodated in the wind function, $f(u)$ ($\text{MJ m}^{-2} \text{d}^{-1} \text{kPa}^{-1}$), by using the empirical functions given by de Bruin and Wessels (1988)

$$f(u) = \frac{0.216u}{\Delta + \gamma} \quad T_w \leq T_a$$

$$f(u) = \frac{0.216u[1 + 10(T_w - T_a)/u^2]^{0.5}}{\Delta + \gamma} \quad T_w > T_a$$

where u (m s^{-1}) is the wind speed at a height of 10 m

and Δ is the slope of the temperature–saturation water vapour curve at air temperature (kPa K^{-1}).

The water temperature at the end of the current time step is then estimated by

$$T_{w,i} = T_{w,i-1} + \frac{W}{\rho ch}$$

where ρ is the density of water = $1000 \text{ (kg m}^{-3}\text{)}$, c the specific heat of water = $0.0042 \text{ (MJ kg}^{-1} \text{K}^{-1}\text{)}$ and h the depth of the water (m).

The procedure is iterated until the difference between estimates of the water temperature at the end of the current time step on successive iterations is less than a pre-set value, 0.01°C in this study, after which the evaporation rate is calculated from the latent heat flux. The initial estimate of the water temperature is set to the value of the water temperature at the end of the previous time step.

The shortwave albedo of the water body is estimated using the procedure of Payne (1972):

$$\alpha = f(\theta, A)$$

where θ is the Sun's elevation and A is the atmospheric transmittance, defined by

$$A = \frac{K^1}{\frac{S_c \sin \theta}{d}}$$

where S_c is the solar constant = $0.0820 \text{ (MJ m}^{-2} \text{d}^{-1}\text{)}$ and d is the ratio of the actual to mean Earth–Sun separation. The appropriate value of albedo is then obtained from a table of measured values.

For clarification, Fig. 1 is a flow diagram of the sequence of calculations involved in the model for a single time step.

3. Test of the models

3.1. Evaporation measurements

Lapworth (1965) reports a remarkable set of measurements carried out between 1959 and 1962 on a pair of reservoirs at Kempton Park ($51^\circ 25' 35''\text{N}$, $0^\circ 23' 46''\text{W}$) in south-east England. Most of the measurements were made on the East reservoir but with measurements

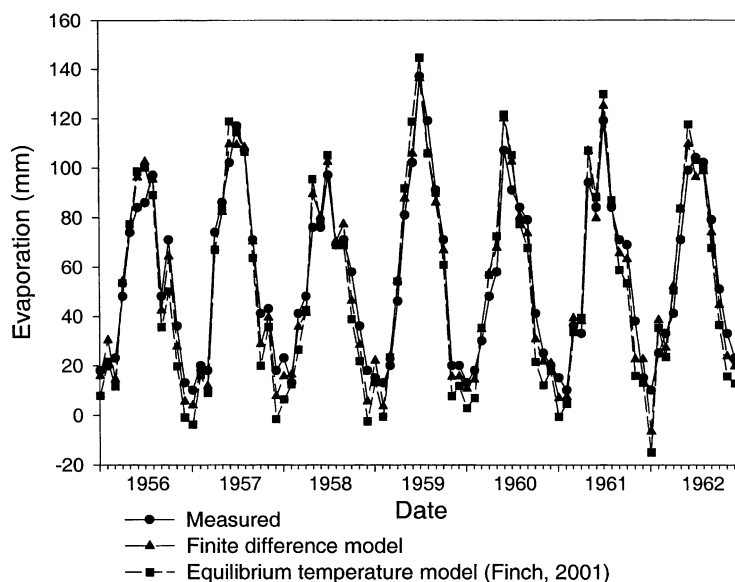


Fig. 2. Measured values of monthly evaporation and those predicted by the finite difference and equilibrium temperature models.

made, for a short period, on the West reservoir for comparison. During the period of measurements, no inflow or outflow occurred, with the exception of a single lowering of the water level in the East reservoir. The East reservoir had an area of 17 ha

and a maximum depth of 7.2 m. Measurements consisted of the water level, which was continuously recorded by a float-operated water-level recorder fixed over the outlet well of the reservoir, and the rainfall which was recorded by a pair of

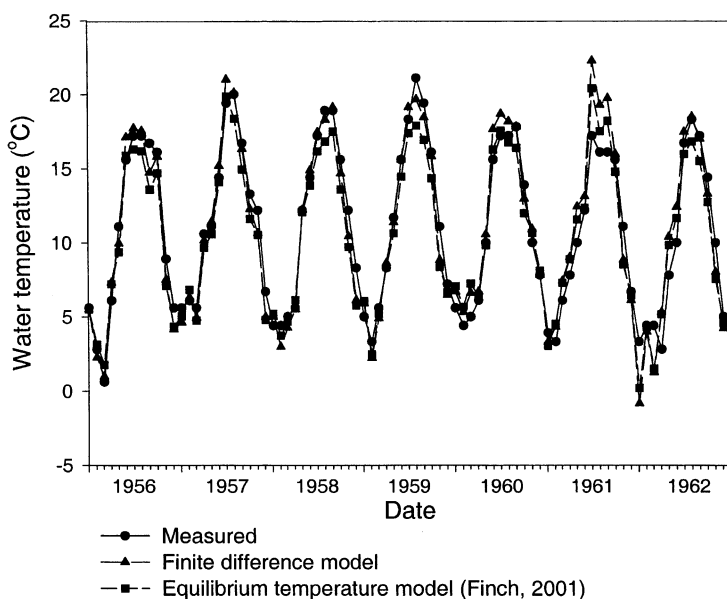


Fig. 3. Measured values of water temperature on first day of month and those predicted by the finite difference and equilibrium temperature models.

Table 1
Error measures (mm) between measured and predicted values

	Mean annual evaporation	Monthly evaporation		Temperature on first day of month	
		RMSE	MBE	RMSE	MBE
Finite difference	–10	7.8	–0.8	1.5	0.0
Equilibrium temperature	–44	11.9	–3.6	1.5	–0.5

raingauges. The water depth was generally between 5.7 and 6.0 m until a single lowering of 1.4 m in July 1959. A mean annual rainfall of 637 mm was recorded over the seven year period. These measurements were used to calculate the evaporation rates using the mass balance method, i.e. the evaporation was equal to the rainfall, plus or minus the change in water level but, to quote Lapworth, “This simple relationship, however, gives no indications of the difficulties which were experienced in obtaining a satisfactory measurement of evaporation”.

The water temperature was recorded near the centre of the East reservoir at approximately weekly intervals and at depth intervals of 1 m. Although Lapworth does not give the values of the depth profiles, he reports that, during the summer months, the water became thermally stratified as the temperature decreased with depth. However, the difference between top and bottom was small, varying between 0.5 and 2.2°C. During the winter, the temperature was generally uniform with depth.

The measurements give a mean annual evaporation of 662 mm, which Lapworth estimates to be within 5% (33 mm) of the true value. Lapworth reports the results as the water temperature, at a depth of 3 m, on the first day of the month and monthly totals of evaporation. The measured evaporation shows a clear annual cycle, Fig. 2, with the minimum occurring generally in January and the maximum in July. About 75% of the annual evaporation occurs during May to October inclusive. There is a significant variation from year to year with the highest evaporation occurring in the summer of 1959, which was a period of severe drought. The water temperature at a depth of 3 m, Fig. 3, shows a similar seasonal cycle, albeit with less variability due to the smoothing effect of the heat storage, and ranges between 0.6 and 21.1°C.

3.2. Meteorological data

The data required to drive the model, for the period 1956 to 1962 inclusive, were obtained as daily meteorological observations of sunshine hours, relative humidity, wind run and average air temperature. There was no meteorological station at Kempton Park and so data were obtained from the station at Heathrow Airport (51° 28' 43"N, 0° 26' 56"W), 7 km from the reservoir. Sunshine hours were not recorded at Heathrow until 1957 and so, the record was extended to include 1956 with data from Hampton (51° 24' 46"N, 0° 22' 26"W).

The procedures given by Thompson et al. (1981) were used to calculate the daily cloudiness factors and the incoming short and long wave radiation, from the measurements of sunshine hours, required by the models.

3.3. Results

The finite difference and equilibrium temperature models were run with a daily time step. The water temperature values predicted by both models simulate the observed seasonal variations well (Fig. 3). However, those predicted by the finite difference model are in closer agreement with the measured values, as demonstrated by the error measures, given in Table 1. The values predicted by the equilibrium temperature model are generally lower than the measured values. The mean bias error (MBE) confirms this and quantifies the systematic error as 0.5°C. There is no discernible systematic error in the values predicted by the finite difference model. The root mean square error (RMSE) is a measure of both systematic and non-systematic errors. The RMSE for the values predicted by the two models are identical, indicating that the non-systematic errors are likely to

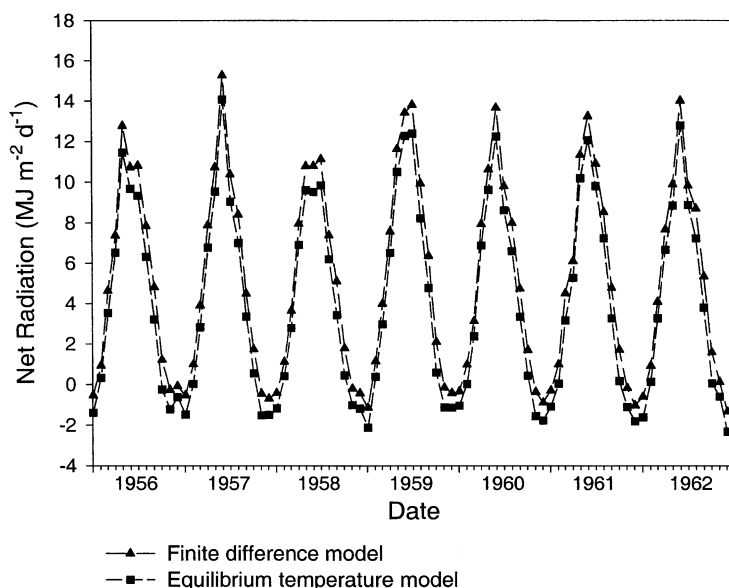


Fig. 4. Monthly average net radiation rate estimated by the finite difference and equilibrium temperature models.

be slightly lower for the equilibrium temperature model.

The daily values of evaporation were aggregated to monthly totals for comparison with the measured values. The low MBE shows that there is little systematic error in the values predicted by the finite difference model, which is also reflected in the low error in the mean annual evaporation. In comparison, the values predicted by the equilibrium temperature model have a systematic bias, which tends to underestimate the evaporation rates. Although the RMSE for the values predicted by the equilibrium temperature model is higher than that for those predicted by the finite difference model, the non-systematic errors in the values predicted by both models are comparable and are within the uncertainty in the measured values.

4. Discussion and conclusions

The values of monthly evaporation rates and the water temperature on the first day of the month predicted by the finite difference model are in excellent agreement with the measured values

and have minimal systematic bias. In comparison, the values predicted by the equilibrium temperature model show a bias towards underestimating both variables. When the monthly average net radiation rates estimated by the models are compared, as in Fig. 4, it is clear that the finite difference model consistently gives higher values than the equilibrium temperature model. The differences between the calculation of the net radiation used in the two models arise from the method of estimating the outgoing longwave radiation and in the heat storage term. Given that there is relatively little difference in the water temperatures predicted by the two models, it is likely that it is the estimation of the outgoing longwave radiation that is the cause of the differences. This illustrates the importance of accurate estimates of net radiation when calculating evaporation rates as, in this study, 88% of the net radiation is converted into evaporation on average.

The differences in the values predicted by the two models are also due to the different wind functions used in the models. The wind function of Sweers (1976) is used in the equilibrium temperature model whilst that of de Bruin and Wessels (1988) has been

used in the model described here. The latter is preferred as it has the ability to make a simple correction for the stability of the atmosphere, which is not present in the former. However, the use of the de Bruin and Wessels wind function with the equilibrium temperature model increases the differences from the measurements (the error in the mean annual evaporation becomes -115 mm) because it tends to give lower estimates of the evaporation rates.

The simpler formulation of the model described here has a lot to recommend it. Although the iteration needed to estimate the change in heat storage increases the number of calculations required, with modern computing power, this is rarely likely to be a cause for concern. In practice, it was found that two iterations were generally required, with a maximum of four on a few occasions.

Both the analytical equilibrium temperature method and the numerical finite difference method give good agreement with the measured values of evaporation and water temperature. This should be expected, as the physics in both approaches is valid. Given the measurement errors in the driving meteorological data and uncertainty in the parameterisation of the models, in addition to the errors in the measurements of evaporation and water temperature, it is not possible to make any distinction between the two models. The advantage of the finite difference method is in its simplicity.

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